RESEARCH ARTICLE OPEN ACCESS

How was Mathematics taught in the Arab-Islamic Civilization? – Part 1: The Pedagogical Principles of Teaching & Learning

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ABSTRACT
No matter how societies have differed throughout history in their values, cultures, politics and systems of life, there has always been a common consensus regarding the importance of mathematics and its role in the social life. Arab-Islamic civilisation -like many others- had a reasonable command of many aspects of mathematics. This was apparent in various applications including not just in everyday life, civil and business transactions, and financial dealings, but also in certain religious rituals.
This paper aims to highlight the important position that the scholars in the Muslim East and West attributed to mathematics, both in terms of teaching and practice. Furthermore, this work investigates the methodology of teaching fractions in the Arab-Islamic society. The latter is found to be one of the mathematical subjects that has been recommended to be taught to learners at their earliest learning stages, especially given the importance of fractions in everyday transactions such as calculating shares of a legacy. The paper also examines some of the educational requirements such as the teacher duties, student's qualities, education stages and what the relevant curriculum recommended by scholars during each stage. Finally, this work briefly highlights one of the contemporary issues which is the difficulty of teaching fractions for young learners and how this was addressed in the past by one of the early Muslim Arab mathematicians through the use of simple poetry.
Keywords – Arabic, Didactics, Education, Mathematics, Teaching.

I. INTRODUCTION

Teaching and education have always been considered as one of the cornerstones of the Muslim society. This is clearly shown by the teachings of the Prophet Muḥammad (PBUH) when He said "I was sent as a teacher", and "My Lord has educated/disciplined me with the best discipline"; While this discipline that the He (PBUH) referred to has been acquired through divine revelation and guidance, for others it would be gained via learning and education. The Islamic history has recorded that the Prophet Muḥammad (PBUH) cared about education since the early parts of His mission, to the extent that establishing the Islamic Governance came as a direct result of Musʿab Ibn 'Umayr's successful educational work in the city of Yathrib, which eventually became known as al-Madīna. In modern terms, the Prophet Muḥammad (PBUH) must have 'hired' Musʿab due to his outstanding teaching skills, and his decision to send him as a teacher to Yathrib turned out to be indeed the right choice. Furthermore, when the Islamic Governance had military activities with its enemies, the authenticated records reveal that the Prophet Muḥammad (PBUH) would not accept ransom money for those literate enemy fighters. Instead, He has set a unique condition for their freedom, which is that they should teach ten Muslim children the principles of reading and writing. The reasoning behind such policies is to spread literacy amongst members of Muslim society and hence to build an individual whose human character is complete with knowledge and distinguished by a sound mind. This soundness can only be perfected by making the individual aware of the realities of things, thus, shaping a personality constantly able to seek what is good and abstain from what is evil, and ultimately achieve the true happiness. Unlike the ultimate satisfaction which is a fruit of embracing knowledge, human ignorance leads to humiliation, abuse and negligence to all wonderful values in life. Equally to literacy, numeracy or mathematics had an important role in the Muslim civilization. This article, outlines that pedagogy, in other words, the educational 'system' that was -in the background- backing the learning of subjects including mathematics.

II. METHODOLOGY OF THE EDUCATIONAL SYSTEM IN ARAB ISLAMIC CIVILISATION

Muslims believe that every human being has been shaped by Almighty God to accept learning by others as well as to be self-taught via listening, speaking, thinking, understanding and learning various fields of arts and sciences. In this regard, *Muḥammad ibn Ibrāhīm Ibn al-Akfānī* (d. 749/1348), the wise science historian and medical doctor, beautifully summarized the difference between knowledge and ignorance in his book *Irshād al-*

qāṣid ilā asnā al- maqāṣid ('Guidance for the Seeker to the Loftiest Goals') by the following statement "….As if that the human being is forced to live as a human creature unless he chooses to learn and avoids the complex of ignorance. If he does so, he will then become a human creature by choice, fully aware of his duties towards his Lord and hence deserving His proximity. Otherwise, if the human being indulges in ignorance, he becomes no different than the rest of animals, if not worse than them…." [1].

Muslim communities in every age and region have recognised how important is to seek knowledge through education and learning. Imam *Sufyān al-Thawrī* (d. 161/777), one of the great scholars of *Fiqh* (jurisprudence) and Hadith as well as a skilled mathematician, highlights the vital role of teachers and educators in society. He reportedly said: "... People need three individuals: a governor ruling by justice, or people will undermine each other's rights; A teacher teaching their children being paid for it, or people will be illiterate; and people need to buy and sell copies of the Qur'ān, or the Book of God will not spread" [2].

Learning is a desire imbedded in all people, thanks to human primordial nature (*fiṭra*). As established earlier, the passion for learning and thinking is what distinguish the human being from the animal. Humans being naturally inclined from infancy to explore the unknown and discover new things, they are inevitably required to start off by tapping back into existing factual knowledge. Learning from those who have gained it already would enable the learner to obtain the skill to utilise factual knowledge when facing new circumstances or conditions. In broad terms, the fruit of learning can be defined as gaining the ability to detect the factual knowledge, understanding its scope, linking between these facts and their associated factors in one side and on the other hand, being precise in attributing the causes with their effects, the evidence with its demonstration, and the argument with its consequence [3] .

In this regard, *al-Jāḥiẓ*, the great teacher and famous writer, that good-quality education is that which sharpens the mind, develops the mind, and balances between the powerful memorisation and the ability of reasoning; he even describes memorisation as being the mind's enemy, ... and the one relaying solely on memory can only be an "emulator" and not a "creative". Reasoning leads to the absolute certainty and confidence,... And extending the use of memorisation would damage the reasoning process. Neglecting the reasoning leads to the loss of justification procedure. On the other hand, neglecting the role of memorisation leads to the loss of these arguments, hence a balance needs to be achieved since the nature of reasoning and memorisation differ [4].

Furthermore, *Miskawayh*'s opinion was that the human being has two kinds of powers: Mental Force and Physical Force, with both of which the human being achieves perfection.

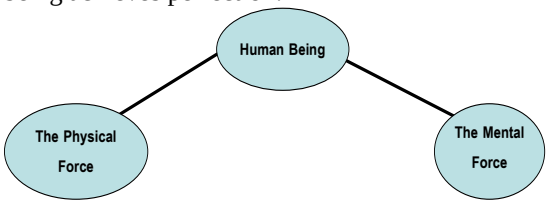

Fig.1 : Miskawayh's split of the human being's powers

Through Mental Force, the learner earns knowledge, while the Physical Force enable the different needs to be organize and put in order. Perfection in knowledge can only attained if the vision is correct, the judgment is true and the opining is straight, with no error in conclusion and without doubts in results. The second aspect of perfection is achieved by the rest of the body parts through the Physical Force. The human actions and decisions would then be delivered according to a systematic approach, promoted to a wider 'civil management' which aims to organise personal as well as collective activities, and ensuring the happiness at the individual and the entire society.

The first part of perfection is theoretical, may be symbolized by an image of an object. On the other hand, the second side is practical and tangible, just like the object itself. Both sides of perfection are hence inseparable. Likewise the object and its image, knowledge and action go hand in hand. Knowledge may be considered as the start of the process, which the action is the goal (end) of it, so knowledge without action is a waste, just like the end without a start is impossible [5] . In addition to the above, *al-Jāḥiẓ* made a number of valuable observations on the need for balance between the senses and the mind. He says in his book *al-Ḥayawān* ('The Animal'): "...Visual observation can lead to errors, people may mistakenly follow what they see, while I would go to where the mind leads. Things may be perceived by two means: that which is visible to the senses and that which is analysed by the mind; the latter is a reliable source of evidence." [6] .

Al-Jāḥiẓ, by this statement, emphasises the importance of acquiring knowledge by analysing the facts in detail. This key methodology is found being adopted widely across the Muslim Society's educational system. For instance, Imam *Abū Ḥāmid al-Ghazālī* (d. 505/1111), in his book *al-Munqidh min al-Dalāl* ('Deliverance From Error'), advised students of knowledge about the need to gain knowledge through analysis, instead of taking facts for granted; he said: …Seek the truth by the analysis

method so that you become a creative in your field, and do not take the blindly-follower approach.... The fundamental objective is to be well versed in the core knowledge of facts. [7] .

Therefore, the effective methodology of conveying educational teachings or writings must be clear and convincing to the learner. This can be achieved provided that a number of conditions are fulfilled, including the clarity of communication to make the learner able to easily understand the purpose of that verbal or written message. In addition, *Al-Jāḥiẓ* insisted that the teacher or author must consider in their communication the receiver (learner or listener)'s intellectual capabilities. He said: The speaker should know the implication of his speech and how to link the levels of listeners and the circumstances of the speech. So, for each situation, there should be a specific communication style; What is needed when addressing people is choosing the right wording according to the relevant meanings, the right expressions according to the circumstances, and being aware of the level of listeners as well as the situation [8] .

However, *al-Jāḥiẓ* had some reservations about two categories of 'knowledge preachers' who will have a negative impact on people, particularly on young individuals. These are firstly the shallowly educated, persons who, when conveying information, they misinform others due to weaknesses in their own intellectual ability; and secondly those highly knowledgeable persons who may deliberately mislead others of inferior intellectual levels. Both of these two types who misguide people, according to *al-Jāḥiẓ*, are considered to be the worst enemies of truth; this led him to describe the appropriate methodology allowing to accumulate other people experiences and to develop one's soundness of mind and perfection in actions. This methodology begins with listening, then escalates to thinking, reflecting, and exploring the details, then eventually implementation in the best manner. He stated that: ... the good listeners amongst people will be the most experienced; the most experienced, in turn, will be those having the brightest ideas; the latter will be those deep thinkers, and those who think deeply about things are those with the largest knowledge. The latter are also the ones with the most correct opinions who make the best actions eventually….., surely this progressing would require a sound mind and a sincere teacher to mentor this process.

This, indeed, is what characterises the methodology of the education system in the Arab-Islamic Civilisation, which examines the individuals (learner or teacher), while taking into consideration the surrounding inner and outer factors, as summarised in the Fig.2 below.

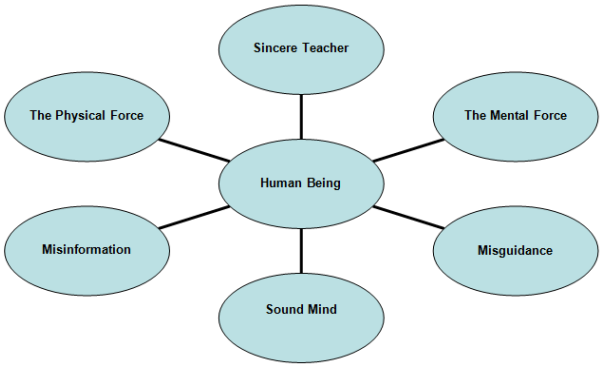

Fig.2 : Inner and outer factors governing the human being's education

Next, this paper describes in details how the above methodology regarded the teacher –student relationship, and discusses the educational subjects being taught in the Arab-Islamic civilisation.

III. THE SUCCESSFUL TEACHER

The importance of good company is indisputable amongst people in the past as well as in present. It is widely recognised that the learner's personality is shaped primarily by his teacher or mentor. This is clearly demonstrated by a number of texts, amongst which is *'Utba ibn Abī Sufyān's* wise advice to a teacher whom he hired to educate his children: "...Prior to teaching my children good manners, let the first thing you begin with is to emphasise these manners in your own conduct. Their eyes reflect your behaviour: they will see good whatever you considers so, they will see bad anything you consider so. Teach them the Book of God (i.e. the Qur'ān) and do not force them while doing so, otherwise they may get bored too soon. Do not be too lenient, so that they will not learn it. Then teach them the finest poetry and the most eloquent way of speaking ... And do not progress from a field of knowledge to the next until you are certain they have fully grasped the first one, because delivering a variety of topics during the same lesson may lead to confusion and lack of focus. Narrate to them biographies of righteous predecessors and morals of sage scholars. Prevent them from engaging in unnecessary talks, and be to them like the doctor who does not prescribe the medication until he knows the cause of illness. Do not rely on any excuse or fear towards me, because I have already ensured all your efforts are compensated…. [9] . These wise guidelines were so valuable that Imam *al-Shāfi'ī* (d. 202 /817) quoted these exact phrases when advising a teacher named 'Abd al-Ṣamad, who was educating the Caliph *Hārūn al-Rashīd's* children, as reported by *Ibn Faraj al-Jawzī* [10] .

In his book *Bidāyat al-hidāya wa ādāb al-ṣuhba* ('The Start of Guidance and the Manners of Companionship'), Imam *Al-Ghazali* (d. 505/1111) defined the key qualities of a good teacher, which

include: tolerance, o remain patient, to sit with respect, avoiding arrogance, modesty and humbleness with people, except with the tyrants as a sign for refusing their oppression. Also a good teacher should appear with a firm personality, avoiding the excessive humour, being gentle with his students, caring for the arrogant learner, correcting the behaviour of those slow learners without anger or frustration. Furthermore, the good teacher should not feel ashamed from saying "I do not know". Questions should be received with full attention and all efforts should be made to understand the query and lay out the best possible answers. He should not hesitate to accept an opposite opinion that is supported by clear evidences, he should adhere to the truth and admit mistakes. H e needs to keep his students away from all misleading information while ensuring they do not seek knowledge with insincere intentions nor to fulfil wrong objectives [11].

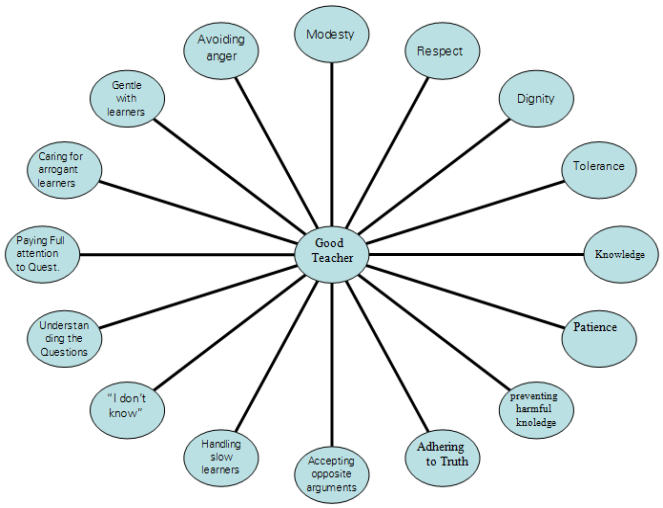

Fig.3 : The Good Teacher's Characteristics

IV. DUTIES OF THE TEACHER TOWARDS LEARNERS

Muḥammad ibn Ṣaḥnūn (202/817), a jurist contemporary of Imam *al-Shāfi'ī*, produced the oldest Arabic book in the field of education titled *Ādāb al-mu'allimīn* ('Rules for Teachers'). He listed in his work a number of key duties of teachers – what would today be called a 'Teachers' Charter', including the following:

The teacher should personally inspect his pupils' daily attendance, and must inform the absentees' guardians…. It is not permissible for the teacher to divert his attention from looking after pupils. He must maintain their safety and regularly check their learning progress.... He has no right to send pupils away to serve his personal needs. He must show a full diligence and high level of dedication to them. He should allocate a session time for reviewing books and encourage pupils to discuss collectively. This is because group discussions enables pupils to enhance their communication, correct their views and sharpen their leadership skills through their attempts to influence each other [12]…. The teacher should be just with learners and not discriminate based on their social or financial status, otherwise he has betrayed his educational mission… The teacher is not allowed to prevent the pupil his food or drink. There is nothing wrong with the teacher disciplining students who neglect their learning or fail to make satisfactory progress. However, disciplining students should not be done while the teacher is angry. Moreover, the teacher shall not be less merciful over orphans and tough over the poor. Corporal punishment may be given up to three times, never using violence or hitting the student's face or head. It should proportionate to the offence, and not be excessive. Students who bullied others may be also disciplined, and the teacher has a duty of care to protect the personal safety as well as belongings of his students. [13] .

V. BREAKS AND STUDENTS' RECREATION

Teacher should release pupils to go home after lessons every day in the afternoon. On a weekly basis, pupils should also be given dispensation every Friday. This was the common tradition of teachers in the Islamic societies, with no comment or text could be found to disagree with this practise. Furthermore, teachers gave additional breaks for their students during the religious feasts of Eid al-Fitr for three days, and during Eid al-Adha between of three to five days. It was also reported that teachers would accord exceptional breaks of one day or more to pupils, but only if authorized by their parents[14] . In addition to these holidays, teachers allocated short breaks during the day, so that pupils can rest from fatigue during their lessons. Teachers also ensured that pupils do not play harmful or tiring games or get used to laziness during their daily breaks. Instead, pupils were encouraged to perform light activities that would replenish their energy and enthusiasm for learning such as walking, physical movements or light sports [15] .

VI. TEACHING MATHEMATICS IN THE ARAB-ISLAMIC SOCIETY

According to *Ibn Ṣaḥnūn*, the teacher should aim to teach his students reading, writing, and spelling and the time. He should also teach them the Qur'ān and mathematics, in addition to the nicely-worded poetry. If possible also, lessons on basic linguistics and techniques of public speaking should be included. As for mathematics, it is recorded in a number of biography and history books that some of the *Ṣaḥāba* (Companions of the Prophet), may Allah be pleased with them, were familiar with the practice of arithmetic. These include: Imam *'Alī ibn Abī*

Ṭālib and his uncle *al-'Abbās* who proposed a method to calculate the inheritance shares using the principle of proportionality which is still applied today. Also amongst these Companions were *'Abd Allāh ibn 'Abbās, Zayd ibn Thābit*, and *Mu'āwiya ibn Abī Sufyān*, for whom the Prophet (SAWS) prayed "Oh Allah, teach him calculus." Even amongst *al-Tābi'ūn* (Followers), the disciples of the Companions, there were also who were proficient in the practice of mathematics and calculating inheritance. *Al-Ḥārith al-Awar* (d. 65/684), who learnt from Imam *'Alī* and *Zayd ibn Thābit*, was a well-known figure in this field and may have been the first scholar who formally taught mathematics at the mosque. A great many students seem to hav e graduated from his circle, including *'Āmir al-Sha'bī*[16]. *Sufyān al-Thawrī* (d. 161/777) was also known to be skillful in arithmetic, besides being a great *Muḥaddith* (scholar of Hadith, the Traditions of the Prophet) and jurist [17] . Amongst the many other *Tābi'ūn* who contributed in the field of mathematics were *Abu Zinād* and *al-Qāḍī Shurayḥ*[18] .

Books for history and biographies also reported that the early Arabic society has practiced a degree of mathematics in different aspects of the daily life, such as in: trade dealings, dividing irrigation water and so on. In addition to this, the principles of geometry have been applied in the areas of construction, architecture and engineering. Astronomy was also practiced to determine directions, especially the *Qibla* (direction of prayer) as well as to estimate distances for travelling purposes. Knowledge of astronomy is demonstrated also through the existence of Arabic words for many planets and stars. This knowledge may have been self-acquired by the Arabic society or been transferred externally due to the contact of Arab traders and travelers with the Babylonian, Indian, Egyptian or Syrian societies.

In view of the need to exercise mathematics, many accounts could be found in which scholars in Islamic society call for learning numeracy. It was narrated by *Ibn al-Jawzī* in his book *Ṣifat al-Safwa* ('Description of the Elite') that Imam *al-Shāfi'ī* recommended his pupils (including *al-Rabī' ibn Sulaymān* and *al-Muzanī*) to implement the work of mind and reasoning while also learning the fields of hadith (the Prophet Muḥammad's teachings), *fiqh* (jurisprudence), *nahw* (Arabic grammar) and mathematics. He said "... Seek help in speech with silence, and in reasoning by work of mind, ... Know that whoever learns the Qur'ān will be highly regarded by people; whoever learns the Hadith, his power of argument is strengthened; and whoever learns *naḥw* will be respected; and whoever learns *fiqh*, his status will be elevated; and whoever learns the linguistics, this personality becomes softer; and whoever learns mathematics, his judgments become sound; and whoever is careless about his wellbeing, his knowledge will never benefit him [19] .

The educator *Miskawayh* affirms the need of teach calculus and geometry to pupils at early ages, after teaching them about *Sharī'a* law and morals. He said "…. The happy complete individual is he who was educated since infancy upon the ethics of *Shari'a* until it becomes embedded in his personality. Then he contemplates in the books of manners until the good morals and values are demonstrated in his behavior. Next, that individual is taught the principles of calculus and geometry so he gets used to speaking correctly and reasoning with accurately. Eventually, that person's ambitions in learning would enable him to progress in knowledge until reaching the maximum possible level. [20] .

As for *al-Jāḥiẓ*, it may be argued that his work of dominantly literature nature may have been the drive behind his tendency to simply teach pupils calculus, without geometry, even though the latter plays an important role in instigating the logical thinking process and developing ones' problem-solving skills, which *al-Jāḥiẓ* also calls for. His views were that the right educational methodology is that which is based upon reasoning and demonstration, and not just on memorisation. *Al-Jāḥiẓ* said "…No need to occupy the pupil's learning by *nahw* (Arabic grammar) except with the basic level that ensures his language is free from obvious errors. Also, the pupil should be taught some calculus, but no need to expose him to geometry, and should be taught how to write essays using plain and simple words ... [21].

However, during and after *al-Jahiẓ*, the field of mathematics prospered in terms of knowledge spread, teaching and writing books in this field. *Abū Manṣūr al-Tha'ālibī* (350-429/961-1023), another prominent literature author, explicitly called to teach mathematics, logic and geometry besides other branches of science. His educational book *Ādāb al-mulūk* ('Etiquettes of Kings') compiled a methodology to equip potential decision makers/rulers with the highest quality education by achieving the finest mental capability, the best physical fitness, as well as endorsing the ultimate level of moral values. He stated "…when the child reaches the school age and becomes ready for learning, it becomes paramount that he receives the best of all the knowledge fields including the *Qur'ān, Tafsīr* (Qur'ān commentary), language and grammar, poetry, mathematics, logic, geometry, astronomy, public speaking and debating, horse-racing, military work and weaponry, military strategy and managing wars, along with writing novels and drafting contracts and legislative texts. Through all of this, this child achieves complete bodily strength with soundness of mind, as well as

reaching the highest education level as soon as he passes his teenage.... [22] .

VII. BRANCHES OF 'KNOWLEDGE OF WISDOM' (PHILOSOPHICAL SCIENCES)

'Knowledge of Wisdom' is referred in Arab-Islamic literature to all branches of knowledge that are acquired through learning and education, such as arithmetic, geometry amongst other mathematics' fields. This is in distinction from the experimental scientific fields such as chemistry, biology etc. The next section examines how this concept was perceived across the Islamic world with a focus on mathematics.

First: in the Islamic East Occident:

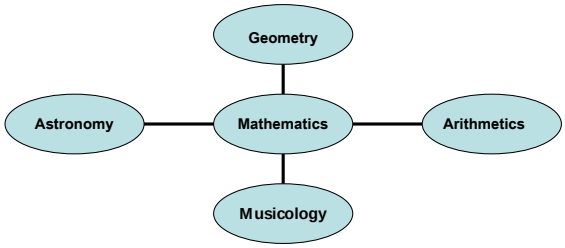

Fig.4 : Branches of Mathematics

Al-Fārābī proposed that the sciences be classified into seven major groups: the science of numbers (calculus), geometry, optics, astronomy, musicology, science of lifting weights, mechanics (physics). On the other hand, the Ikhwan al-Ṣafā' (Brethren of Purity) in their letters favoured a narrower split to mathematics of four sections: Theory of numbers, geometry, astronomy, music (composition, not the instrumental aspect).

Secondly: in the Islamic West Occident:

As soon as Islam spread west of the Arabian peninsula, educational institutions begun to appear, starting by setting up Katatib (preliminary form of schools for pupils) then large knowledge centres appeared as a natural progression following the foundation of cities such as Al-Kairawan. The latter was considered 'the mother of cities' across the Western region of the Muslim world. At an early stage, a number of great scholars in various scientific fields. In mathematics, Shaqran ibn Ali (d. 185/802) became quickly famous after writing a book in calculus of significant scientific value that reached Al-Andalus, and continued being teaching until the 6th/11th century, as established by Ibn *Khayn al-Ishbili* (d. 575/1179) in his curriculum.

Other authors appeared in the field of calculus and inheritance calculations such as *Yahya Al-kharraz* (134/237 A.H. - 1100/852 A.D.) as well as Muḥammad Ibn Sahnoun, who stressed in his writing Adabul-Mu'alimeen (Etiquette of Teachers) on the importance of teaching mathematics to children. He was also supported by *Abulhasan Al-Qabisi* (d. 403/1012) who, like *Ibn Sahnoun*, highlighted the need of teaching calculus as outlined in his book *Arrisalah Al-Mufasalah Li Ahwal Al-Mouta'alimeen wa Al-Mou'alimeen* (The Detailed Letter to Learners and Teachers). Furthermore, Ibn *Al-Monomer At-Tarabulsi* (d. 475/1033) also agreed with other scholars as outlined in his work: *Al-Kafi Fi Al-Hisab Wa Al-Faraid* (The Sufficient in mathematics and calculating the inheritance) [23] .

In Al-Andalus, teaching in the early stages was limited to the field of Shari'a law and the associated sciences of language and literature. However, after a moment of debate, students opened up to the various fields of sciences, producing a number of brilliant figures who appeared in various branches of knowledge. In the 'Andalusian' biographies, names were be found of some early scientists who contributed in founding the Islamic sciences and mathematics in this region, such as *Abdul Rahman Al-Habli* (d. 100/718), who was amongst the early scholars who taught in Al-Andalus and previously in *Kairouan*. He left several students, including *Omar Ibn Sharahil* who was appointed by the Caliph *Abdul Rahman* the First as a judge over Córdoba. Amongst these brilliant individuals was also *Hanash Al-San'aani* (d. 100/718), the famous architect and mathematician, as well as *Yahya Ibn Yazid* (d. 142/759), who was appointed by the Caliph *'Umar ibn Abdul Aziz* as the supreme judge over the entire Al-Andalus district.

Abdu Al-Malik Ibn Habib (177-238/793-852), became the jurisprudence scholar of Andalusia after travelling to seek knowledge in the Eastern orient (in 208/823) and returned to Al-Andalus after acquiring a considerable level of knowledge. He reported some problems from *Al-Khawarizmi* (d. 235/850)'s algebra book (*Algabr wa Al-Moukabala*). *Ibn Habib* inspired a number of pupils, including *Muḥammad ibn 'Abd al-Barr al-Kala'ei* (d. 283/896), who may be considered to be one of the early mathematicians in Islamic Spain, since no earlier person from that land can be traced in historical or biographical sources.

This well-founded learning environment in Andalusia flourished during the reign of *Muḥammad I* (222-285/836–898) and extended till the 3rd/9th century. During this period, this region has seen the launch of a wide cultural 'renaissance'. It was a scientific as well as a political competitive development to that which was taking place in Baghdad and led by the Abbasids in Iraq. A number of prominent mathematicians appeared in Spain, such as *Aḥmad ibn Ibrāhīm Ibn al-Qurṭubī* (d. 290/902), then followed by emergence of other well-known scientists in various scientific fields, particularly in those which were associated to some

Islamic teachings or rituals, such as calculus, inheritance, geometry and astronomy [24] .

This categorisation of fields of learning can be found in *Ibn Ḥazm al-Andalusī's* (d. 455 /1063) treatise *Marātib al-'ulūm* ('Stages of Sciences'), as well in *Sa'īd al-Andalusī's* book *Ṭabaqāt al-Umam* ('Classes of Nations'). It can also be found in other works by Maghrebi scientists who elaborated on the classification of the scientific fields and highlighted the status of mathematics in the Islamic society. These include *Ibn Khaldūn in al-Muqaddima* ('the Introduction'), and Ibn Ḥaydar in his commentary on *Ibn al-Bannā's* book *Talkhīṣ a'māl al-Hisāb* ('Summary of Mathematical Operations').

VIII. THE EDUCATIONAL STAGES AND RELEVANT CURRICULUM FOR EACH STAGE

Instruction materials do not appear to be just a matter of the teacher's choice. Instead, greater scholars have proposed some forms of guidance or curriculums which appear to be implemented by the wider teaching community. For instance, *Ibn Hazm* has suggested a practical learning programme that he illustrated his work with. It was presented in details according to the learning stage or level, the subjects taught in each stage and listing the recommended textbooks. In summary, he states that at five years old, the child generally becomes fully able to understand messages and likely be able to formulate meaningful answers. At this age, guardians should appoint an educator for teaching him writing and reading. The child, during his process of seeking knowledge, progresses through the following seven stages.

Stage One: Literacy Basics

This is to learn writing and reading, by learning spelling (i.e. writing and composing words from the alphabetic letters); acquiring fluency in writing clearly and correctly; and gaining competency in reading any kind of book. During this period, it is recommended for the child to learn the Qur'ān, for the purpose of improving the learner's reading and practising correct recitation, in addition to benefiting from its exalted meanings and valuable morals. After reaching a confident level in writing and reading, the pupil should progress to learning the language and its grammar in detail.

Stage Two: The study of linguistic sciences: grammar, language and poetry

This is to master the language rules in order to conduct effective conversations. To gain a good understanding after reading knowledge books. To become proficient in poetry. Since poetry was an essential communication tool in the Arabic culture, Ibn Hazm advises on learning famous poetry texts, particularly that which addresses good wisdoms and morals, including that of Ḥassān ibn Thābit, Ka'b ibn Mālik, 'Abd Allāh ibn Rawāḥa, Ṣāliḥ ibn 'Abd al-Quddūs. Furthermore, poetry to be learnt at this stage should not call to immorality, violence, corruption and the squandering of money on negative purposes, or that which calls to criminality or which facilitates for the learner to leave family life live in exile. Also to be avoided are satirical poetry (hijā), because it promotes negative feelings, encourages breaking social ties, and spreads disrespect. As for poems of praise or mourning, these are considered to be among the permissible things that should be avoided, according to Ibn Ḥazm.

In terms of textbooks in *Naḥw* (Arabic grammar), *al-Wāḍiḥ of al-Zabīdī* and *al-Mūjaz* (The Summary) of *Ibn al-Sarrāj* are been recommended. The latter book is also to be used as an introductory level to *Sibawayh* books.

In order to gain the basic knowledge in the Arabic linguistics, *Ibn Hazm* advises to use two books: *al-Gharīb al-Musannaf* (Classified Rarities) of *Abū 'Ubayd*, and *Mukhtaṣar al-'Ayn* ('Summary of The Source). For detailed reading, he even recommends to study additional references including *Thābit's Khalq al-insān* ('Human Creation') and *al-Farq* ('Discrimination'), *al-Mudhakkar wa al-mu'annath* ('Masculine and Feminine') by *Ibn al-Anbārī, al-Mamdūd wa al-maqṣūr wa al-mahmūz* ('Forms of the sign Hamza') by *Abū 'Alī al-Kālī*, and *al-Nabāt* ('Plants') by *al-Dīnawarī*.

Once the pupil becomes skillful with the language, the next step according to *Ibn Ḥazm* would be then to study 'the science of numbers', i.e. mathematics.

Stage Three: Study of Mathematics

This includes mastering the following techniques: addition, subtraction, multiplication, division, fractions, surface areas, numbers' properties, planetary rotations, solar and lunar eclipses, times of day and night, marine tidal movements (transgression and regression effects).

To gain a grounding in this field, the works of Euclid (fl. 300 BC) and Pseudo-Ptolemy's *al-Majistī* were recommended. Euclid's book should be studied attentively as it helps to understand properties of various natural phenomena such as earth specifications, planets' positions and dimensions. *Ibn Ḥazm* also not only draws the attention to the information listed by Euclid, but also to demonstrations listed in the book as evidences against his claims.

By reading *al-Majistī*, the student also learns about the eclipses, dimensions of cities, estimating times during the day or night, tidal movements and how to establish astrological orbits for the various planets and stars. *Ibn Ḥazm* also mentions other

purposes of learning this field such as organising irrigation, lifting heavy objects, engineering construction and building 'intelligent' machinery.

Stage Four: Study of logic and natural science.

This involves learning about the different natural species, weather phenomena, animals and vegetation, minerals and anatomy.

Stage Five: Study of History

This involves learning about ancient and the recent historical events from its reliable sources, the most authentic of which is Islamic history, and then to learn about other civilisations and nations such as the Israelites, followed by the Romans then the Persians.

Stage Six: To learn how to evaluate the proofs and evidences on matters of faith.

The issues to be addressed include the following: Had the world a beginning or has it always existed? Does the universe have a creator? Is the creator one or multiple deities? Is prophethood possible? Also included are reviewing the differences between other faiths and reviewing the prophethood of the Prophet Muḥammad (PBUH).

Stage Seven: Study of the bases and branches of Sharī'a law

This also includes the knowledge of Qur'ān, Hadith, jurisprudence and philosophy.

Ibn Ḥazm states that for each nation, science may be divided into seven sections, regardless of time and locations of nations, beginning with the science of religion (every nation must have an ideology or belief that needs to be outlined for every member0; then the science of history; then the science of language. The nations of the world differ in the content or features of each one of these three categories. The remaining four types of science, by contrast, are agreed upon by all nations. These are the science of numbers (mathematics), astronomy, medicine, philosophy (understanding the reality of things, objects or humans) and divine sciences (metaphysics).

Ibn Ḥazm affirms that everything that has any value is considered part of knowledge. This includes the various skills such as business trading, sewing, knitting, marine sailing, farming lands, planting and gardening, building and construction out of many other competencies that can be learnt to benefit people. These branches of sciences are also interlinked and one field is indispensable of the other. Therefore, the student must not criticise what is unfamiliar with, which could be a sign of deficiency in him. Likewise, the student should not be overconfident as to what he has learnt, which could be a sign of arrogance or lack of modesty.

IX. CHARACTERISTICS OF THE STUDENT OF KNOWLEDGE

Here in this section, a number of recommended values which distinguish the diligent student's behavior are listed according to *Ibn Ḥazm*:

- The student's character should be free from negative habits or blameworthy qualities.
- The intention from learning should be to lift one's status from ignorance.
- Not to acquire knowledge for the purpose of seeking fame or to be praised by others.
- The student must have a good level of intelligence, understanding, perseverance in research, patience, commitment, and readiness for fatigue and spending money.
- Not to be distracted by other worldly matters, especially while away from family and friends.
- To behave with the teacher with modesty and humbleness.
- To comply with the teacher's advice, just as a patient complies with his doctor's recommendation.
- Extensive reading. There is beneficial information in every book. Books are the means of storing information; without them, knowledge will be lost, since there is no way to memorise all fields of knowledge. Also without reference to books, there would be no way to distinguish between explanations given by the knowledgeable and those given by the ignorant.
- Knowledge should be acquired from multiple sources, not from one opinion only.
- Learning should not be focused on one field only, while remaining ignorant about everything else. Given that the various parts of sciences are interlinked with each other, the student should take from each field to the best of his ability.

Ibn Ḥazm's programme was found to be fruitful across various social levels, at least for sons of princes and wealthy families. Shortly after implementing this programme, great scientific personalities started to shine in the cities of Andalusia, and the same effect was simultaneously observed in North African cities such as Fez, Marrakech, Tilimsān (Tlemcen), Bajāya (Bougie) and Tunis. These personalities have excelled in the scientific fields of reasoning (mathematics etc) and transcriptions (Islamic jurisprudence etc).

The cities of Maghreb embraced several renowned scholars including *Muḥammad ibn Ibrāhīm al-Ābilī* (681-757/1228-1356), a student of *Ibn al-Bannā al-Marrākushī* (d. 727/1321) known as the greatest mathematician of the Maghreb. *Ibn al-Bannā* was also reportedly the teacher of other leading scholars such as *Ibn Khaldūn* the philosopher of history; *Sa'īd al-Uqbānī*, the

mathematician and judge; *al-Sharif al-Tilimsānī*, a jurist well versed in geometry; and many others.

In addition to their specialist research, these scholars also contributed to the methodology of teaching. *Al-Ābilī* proposed practical solutions to some common educational issues, such as the following... 'When reading a book, the student of knowledge should not contemplate on the misunderstood parts of this book until he has completing reading the entire book. This is because the first parts of any book are related to the subsequent ones; after relating the early parts to the later ones, their understanding should be complete. However, when the student occupies himself with the unclear passages, his reading will stop and he will not be able to complete the book. Reading books in full is one of the requirements of knowledge. If one does not finish reading books, or reads only a few parts, his understanding will be incomplete and he cannot be eligible to teach it to others either [25] .

Furthermore, the major cities of Andalusia witnessed the emergence of great scientific figures who excelled across many scientific fields including mathematics. These include *Abū al-Qāsim al-Qurashī* of Seville (d. 580/1184 in Bijāya), a prominent mathematician and scholar in jurisprudence and education who produced the best commentary on *Abū Kāmil al-Miṣrī's al-Jabr wa al-muqābala* ('Algebra'). Another famous scholar in Andalusia was the *Qāḍī* (judge) *Abū al-Qāsim al-Hūfī* (d. 588/1192) author of the work on *al-Farā'iḍ* (the division of inheritances) which is deemed especially valuable owing to its proficient applications of calculus and algebra principles.

To demonstrate additional fruits of *Ibn Ḥazm's* programme, it would be worth highlighting a scholar contemporary with *al-Qurashī* and *al-Hūfī*, namely *al-Qāḍī Abū Bakr Ibn al-'Arabī* of Seville (d. 543/1147). His position amongst the leading Muslim jurists is undisputable. However, his contribution in the fields of mathematics was almost unknown until recently. A text written by *Ibn al-'Arabī* himself and examined by the author of this paper reveals for the first time that this scholar did study algebra as well as geometry during a process of seeking knowledge. It may be argued that this process may have been set up in accordance with *Ibn Ḥazm's* programme. Ibn *al-'Arabī* says about himself[26] : '... Out of God's bestowed mercy on me, when I was in my early childhood, my father arranged a personal teacher to teach me the Qur'ān until I learnt it when I was nine years old. Then he provided me with three teacher: one to master the Qur'ān rulings, the second to teach me the Arabic language and the third to teach me mathematics. So, as soon as I reached my sixteenth year of age, I became a skillful reader of Qur'ān and I acquired a firm foundation in Arabic by reading: *al-Īḍāḥ* ('The Explanation') by *al-Fārisī*, *al-Jumal* ('Sentences'), the *Kitāb of al-Naḥḥās*, and *al-Uṣūl* ('The Fundamentals') by *Ibn al-Sirāj*. I also heard *Kitāb al-Amālī* ('The Aspirations)' ..... In mathematics, I practiced the mathematical operations, algebra and the inheritance calculations. Then I read Euclid (c. 300 BC)'s book then learnt about geometry of Apollonius (c. 100 A.D.) and I applied the three astronomical tables (used to determine the cosmological schedules movements). I also used the astrolabe, and I studied the projective points' configurations (meaning, perhaps, projective geometry).

These three teachers used to rotate on my lessons from morning prayers until the *'Aṣr* time (mid-afternoon), then left me until the next day for my own time which I used for rest, reading, revising or contemplating on beneficial information learnt…." .

Calculus (numbers' theory) was considered at the time to be the second branch of mathematics, after arithmetic. *Ibn Khaldūn* in *al-Muqaddima* ('The Introduction') expressed this by saying [27] : 'Calculus (number theory) is the process of computing numbers by annexation and separation ... this annexation and separation occurs either in the correct numbers, in fractions, ... or in the roots. This work of manipulating numbers is required by people to facilitate their various transactions; it has been a subject of publication and study; people are eager to learn it as well as to teach it to their children. It has even been recommended that this subject be learnt prior to any other kind of knowledge, due to its clear-cut methods and well-structured demonstrations. The student who learnt it first would develop a luminous mind that is trained to be right and accurate. In addition to this, it may be argued that by acquiring numeracy first, the correct facts and the analytical spirit of calculus would be impressed on students' personality by making them accept nothing less than the truth in all matters and making this a permanent habit in all aspects of life.

One of the branches of the science of numbers (i.e. mathematics) is algebra, which is the methods to extract the unknown parameter out of the known or given ones and based on a relation governing this link. The unknown parameters are of three types, depending on their multiplications (in the equations: the number (an absolute value), an unknown variable (x), a squared variable (x^2).

Another branch of mathematics is 'transactional computations', referring to the application of calculations to daily transactions such as inter-city trading between cities and calculating land areas or *zakāh* amounts (obligatory alms). This involves the application of mathematical tools such as known and unknown parameters, ratios and fractions, natural numbers, and roots. The person becomes expert in

this field through continuous practice on solving various problems and by training himself using different exercises until this skill is gained and established.

The branches of mathematics also include *al-Farā'iḍ*, which is defined as a set of methods and techniques by which a deceased person's wealth is apportioned between the heirs in accordance with *Sharī'a* law. As in other branches of mathematics, this is done mainly by using fractions, natural numbers, roots derivations as well as calculating unknown values from known parameters.

X. ADDRESSING TEACHING DIFFICULTIES THROUGH EDUCATIONAL TEXTS

Nowadays, learning mathematics is widely viewed as one of the least favourite subjects at school owing to its perceived complexity, particularly for subjects such as fractions and ratios. It is interesting to note that this was also the case in mediaeval Arab-Islamic society. However, there is evidence that mathematicians during that time managed to teach a large number of students using a number of resources available to them, employing simplified methodologies and language to conveying the lesson content.

To illustrate the pedagogical techniques found in school textbooks of the past, we conclude this article with an extract from one such text. Mathematical theories or rules phrased with complicated wordings were simplified and 'unappealing' texts were replaced by thematic didactic poems. This method became widely used in most subjects, especially after the era of scientific prosperity in the Muslim world (i.e. after the 6th/12th century). Poems on calculus, algebra and geometry were composed by mathematicians who were also masters of literary Arabic. One such scholar was *'Abd al-Raḥmān al-Akhḍarī* (9th/14th century), a renowned jurist, linguist and educator, who lived in the region of Biskra in Algeria. He composed several educational mnemonic poems, including the text translated below which summarises the principles of arithmetic and includes a chapter on ratios and fractions.

Section One: Numeric Characters (integers)

Its characters are well known: from one to nine are to be mentioned;

and they made the zero as a symbol of nought, which is round as a clear circle.

Numbers have four groups, the first is the class of units,

then the class of tens followed by the hundreds, then the thousands are to be called out;

then from the class of thousands, the subsequent figures vary according to the class of units.

Section Two: Addition

Addition is joining one number to the other, to count it up as a single word.

The class of units are arranged below each other, then likewise for the other classes;

add each figure to that scribbled under it, then look at the total summation.

If the sum was nine or less, put this number above the line of numbers which produce it;

if the sum was higher than nine, write the extra under the next line of numbers,

then count it with that line of numbers, and the end result (of the addition) is what is above the line.

If you add a number with a zero, write that line above so that you know;

and if you add two zeros together, write above only one of them.

If the numbers going down have been repeated,

Just add them to the other numbers, without any change to the number. Such is the rule.

Section Three: Subtraction

Subtraction is dropping few from many, and is done according to six principles;

if you take something out of a bigger value, this is a straightforward subtraction.

Adjustments are in two ways. If the upper number is a zero or is smaller than the lower one,

increase the upper number by ten, then subtract, while adding one to the lower number in the second row;

and zero is the result of subtracting a number from itself, just like taking a zero from another zero.

If the zero is in the lower row, the result (of the subtraction) is the number in the upper row.

All these steps will lead to a final result, provided that (the numbers) in the upper row are higher than those underneath.

Section Four: Multiplication

Know that multiplication involves duplicating a number, a number of times equal to the other number,

So write these two numbers in two rows, and arrange the digits below each other.

Each figure (from below) is to be multiplied by every other digit from the upper row,

then the result of each step is to be written above the lines,

progressing from one digit to the next (in each line).

Then the results are to be added up with one another.

If one is multiplied by one, the result is always one, no more;

and if you multiply one by any number, the result is that same number;

be content if you get nothing after multiplying a zero by itself or by any other number."

This poem also covers other mathematical operations, including division: numerical problems and basic equations; ratios and fractions; and operations using ratios. See the full text, in Arabic, given in Appendix 1.0. Then the sections gradually progress from simple concepts to more complex mathematical principles and methods. Such was the methodology recommended to students of knowledge.

XI. CONCLUSION

As highlighted in this paper, teaching and learning played an important aspect in the Islamic Civilisation of the medieval era. This was demonstrated through the huge work of scholars who accurately narrated to us the pedagogical environment at the time, including the qualities of good teachers, educational methods implemented by the teachers. The paper also listed samples of the curricula taught to students and assessed up on before they would pass from one level to a higher one. The learning objectives offered to developing both intellectual as well as physical skills of students. Attaining these objectives cannot succeed without the role that the educators and teachers, a role that we demonstrated the educational system very much acknowledged. Furthermore, we discussed the specific criteria upon which their teaching quality was benched marked to measure the knowledge delivery process. Under this background environment, teaching mathematics was practiced in various parts of the Muslim World and lead to imminent mathematicians to emerged from these schools and became famous teachers themselves. The latter have also compiled their knowledge in books of various levels of complexity and even learnt how to adapt methods of addressing various learning issued faced throughout their teaching experience, including how to teach difficult mathematical subjects such as fractions, etc. Further work is still required to focus on specific elements of mathematics and how they were taught in the medieval time by the Arabic mathematicians. The latter have surely used solutions that would serve as a guidance to our contemporary teachers to overcome topics that are still found relatively difficult. For this purpose, the authors will discuss further in the second part of the this paper the aspects of teaching fractions to young learners and examines what pedagogical techniques were implemented by teachers back then.

Acknowledgements

This work was supported by Professor A. Khelladi, director of the Operational Research Lab at the USTBH, Algeria. The author would like to thank him for his valuable contribution and guidance in the course of this work.

Also the author expresses his appreciations to Dr Mohamed Issa Whelly, Curator at the British Library, UK. Finally, the author is deeply indebted to all people who offered support to complete this paper. Without them, this work would have been certainly harder to complete.

Notes and abbreviations:

A.H.: Anno Hegirae (in the lunar year after the Hijra (Year 1 A.H. began in 623 A.D.).
A.D.: Anno Domini (in the year of the Christian era, according to the Gregorian calendar).
B.C.: before the birth of Christ
d.: died
n.d: no date stated on the published work

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Appendix 1.0 – The Full Poetry (in Arabic) of Al-Akhdari on Mathematical Operations

قال الشيخ الفقيه العالم العلامة أبو زيد سيدي عبد الرحمن الأخضري رحمه الله تعالى :

الباب الأول في حروف الغبار

حروفه معلومة مشهورة ..... من واحد لتسعة مذكوره
وجعلوا الصفر علامة الخلا.... وهْو مدوّر كحلقة جلا
وأربع مراتب الأعداد ..... أولها مرتبة الآحاد
والعشرات بعدها المئونا ..... من بعدها الآلاف يذكرونا
ومن هنا تبدُّلُ الأعدادِ ..... وترجع الآلاف كالأعداد

الباب الثاني في الجمع

الجمع ضمُّ عدد لعددِ ... لكي تعُدُّه بلفظ مفردِ
فتُجمَع الآحادَ للآحاد ... وهكذا الباقي على التمادي
ضف كل رتبة إلى الموضوعْ .... من تحتها وانظر إلى المجموع
فإن يكن تِسعا فأدنى فلتضع ... جملته فوق الذي منه اجتمع
وما يكون زائدا عليها ... فانزل به تحت التي تليها
واجمعها مع أعدادها بالضبط.. فخارجٌ ما كان فوق الخط
وإن جمعت عددا لصفر ... فاطلع إذاً بعدد لتدري
فإن جمعت ههنا صفرين .. فاطلع بواحد من الاثنين
وإن تكرر الذي قد نَزَلاَ .. به لكون الجمع قد تسلسلاً
فاجمعه مع أعداد ما به عرى .. من دون تغيير له كذا جرى

الباب الثالث في الطرح

الطرح إسقاط قليل من كثير... وهو على ستة أقسام يصير
فإن طرحت القدر من كثير... فالطّرح فيه واضح التقدير
والحمل في قسمين إن صفر..... علا أو كان الأعلى أدن مما سفلا
فاحمل عليهما بعشرة وافية.... واطرح وأدخل واحدا في الثانية
والصفر كاف إن طرحت العددا..... من مثله كالصفر من صفر بدا
وإن يك الصفر الذي من أسفلا... فاقنع إذاً بعدد قد اعتلا
وكل ما ذكرت من أقسام ... فيما عدا الآخرِ ذي التمام
لأنه حتما يكون أكثرا... من الذي من تحته قد شهرا

الباب الرابع في الضرب

اعلم بأن الضرب تضعيف العدد... بقدر ما في آخر من العدد
فاجعلهما سطرين كلّ مَرتبة... مقرونة بأختها مرتّبه
فكل رتبة لأعلى تنسب ..... في رتبة الآخر طرا تضرب
واحسب من المضروب للمضروب فيهْ... والترك لا من واحد تكن نبيهْ
ولتجعل الخارج فوق الأسطرِ.... بقدر ذلك الحساب الأشهرِ
ويُجمع الخارج ثم يُجعلُ ... من فوقه وبعد ذاك يُفعلُ
وإن ضربت واحدا في واحدِ ..... فواحدٌ يكون دون زائدِ
وإن ضربت ذاك في الأعداد..... فقَدْرُ ما فيها من الآحاد
فاقنع بصفر إن ضربت الصفر في.... نظيره أو عددٍ فلتقتفي

الباب الخامس في القسمة

وعمل القسمة في الحساب .... من أحسن الفصول والأبواب
فلتجعل المقسوم فوق الآخر.... وتجعل الإمام فوق الآخر
ولا يجوز أن يكون الأكثرُ..... تحت الأقل منه بل يقهقر
ثم تروم عددا يُضرب فيهْ ..... تُفني به الذي عليهْ
وما بقي فضعه فوق ذاكَا... وقهقر الإمام من هناكَ
فإن تعدى رتبة فلتجعلا ..... صفرا قبالة المعدّى أسفلا
وافعل كما ذكرته إلى التمام .... فخارج ما تحت ذلك الإمام
وما بقي من الكسور يطلبُ..... فوق الإمام ثم منه ينسبُ

فصل

وإن تشأ فتأخذ الوفقين ... واعمل عليهما بغير مينِ
أو حُلّ مقسوما عليه واقسما ... على أئمة له لتعلما
أو تقسم المقسوم بالتفصيل... وتجمع الخارج بالتعديل

الباب السادس في التسمية

تسمية نسبتك القليلا .... من الكثير فاعرف التمثيلا
فالقِهِ أئمة لتقسما..... من بعد أن تحلّه فلتعلما
والبدء في تنزيلها بالأكبر... والبدء في قسمتها بالأصغر
وما بقي من الكسور يرسم... فوق الإمام ثم منه يعلم
واقسم على الذي يليه ما خرج .... وافعل كما ذكرته فلا حرج
فكل ما على الأئمة تصبْ ..... هو المسمى منه مثل كسر ينتسب
وإن تشأ فانظر إلى الأوفاق... واعمل عليها عند الاتفاق

فصل في حل الأعداد

قد ذكروا لحله مقدمةْ.... لازمة لكل من تعَلَّمهْ
النصف والعُشْر مع الخمسِ لما ... الصفر في أوله تقدّما
وإن يكن مفتتحا بالخمسةْ.... فذاك ذو خُمسٍ تفهّم أسّه
واعلم بأن جملة الأعداد..... مقسومة للزوج والإفراد
وليطرح الزوج بطرح التّسعة ..... مع الثمان ثم طرح السبعة
فإن طرحته بتِسعٍ فالسدُسْ... له وتسع مع ثلث فاقتبسْ
وحيث ستٌّ أو ثلاثٌ عبرا ... فالسدْس والثلث له قد شُهِر
وإن بقي ثلاثة فالسدس له ..... والثلث أيضا فادر تلك المسألة
واطرحه إن بقي بعد ذلك .... طرح الثمان تتبع المسالك
فالثمن والربع له إن انطرح .... وإن بقي ربع فربع اتضح
وإن بقي ما عدا ما قد شرح... فاطرحه طرح سبعة إن انطرح
فذاك ذو سُبع وإن لم ينطرح... فذاك إلا النصف فردا يتضح
وفردها بطرح تسعٍ يطرحُ ... وطرح سبعة بذاك يوضحُ
فإن طرحته بتسع فالتّسعْ... له وثلث فتفهّم واتبع
وإن بقي ثلاثة أو ستةُ .... فذاك ذو ثلث فحسبُ يثبتُ
وإن بقي غير ما قد ذكرَا .... فاطرحه طرح سبعة واعتبرا
فإن طرحته بذاك الطرح ..... فذاك ذو سُبعٍ تفهّم شرحي
وإن يكن لم ينطرح فهو الأصم... قسِّم من أجزائه ما قد علم

الباب السابع في الاختبار

فيه ثلاثون بتيا أولها

الاختبار آلة قد علما .... يفيد في جميع ما قد تقدما
فاختبار الجمع ذو وجهين .... إما بطرح أحد السطرين
من خارج فاعلم ويبقى الآخر .... فواضح بيانه وظاهر

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باب الكسور ويشتمل على فصلين

الفصل الأول في أقسامها

والكسر منه مفرد ومختلفْ ..... مبعّض منتسب كذا عرف
فذو اختلاف مثل ثلث وربُعْ... وذو انتساب مثل خُمْسٍ و سُبُعْ

بضرب ما على الإمام الأول ..... في كل ما يليه فليكمّل
وذو انتساب كاختبار النسبةِ..... وقد مضى تقديره بالجملةِ
والمختلف بضرب بسط ما قُصدْ... في كل ما من تحت غيره عُهِدْ
وضرب بسط ذاك في إمام ذا .... ويحمل المجموع فافعل هكذا
وإن يكن هنا صحيح يدرَى ... كأنه بسط الكسور شُهِرَا

الفصل الثاني في أعمال الكسور

وإن ترد ضرب الكسور فاضربَا ... البسط في البسط وكن مرتبَا
فقدم الكبير في الأئمةِ .... يبدو لك المطلوب بعد القسمةِ
ووصف قسمة الكسور هكذا ... بضرب بسط ذاك في إمام ذا
والعكس واقسم خارج المقسوم .... عن خارج الإمام كالمعلوم
وهكذا تسميةُ الكسورِ .... ويقسم الأدنى على الكثيرِ
ومثل ذاك الجمع لكن تُجمعُ..... والخارجات بعده توزَّعُ
والطرح يطرح الأقل منهمَا .... من الكثير فيه ثم تقسمَا
واختبر الطرح بطرح بسط مَا ..... بدا وسطريه كما تقدما
وخارجاً كالمقسوم في ..... جمعٍ وقسمةٍ ونسبة تفي
يُطرح بسط ما بقي وما ظهرْ... من ذيْنِك الشطرين طرحا يُختبَر